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ARE FINANCIAL RETURNS REALLY NORMAL?

A Statistical & Non-Parametric Analysis of S&P 500 Returns

Empirical evidence of non-normality, heavy tails and the implications for financial risk

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1. RESEARCH QUESTION

Do daily S&P 500 returns follow a normal distribution, or do they exhibit statistical characteristics that require alternative methods for financial risk analysis?

2. OBJECTIVES

- Examine distribution of daily S&P 500 returns
- Test normality using graphical & statistical methods
- Measure skewness, kurtosis & tail behavior
- Estimate empirical distribution (KDE & ECDF)
- Compare empirical vs theoretical quantiles
- Identify extreme return observations
- Discuss implications for financial risk modeling

3. DATA & METHODOLOGY

Data: Daily S&P 500 Index (2010 – 2026)
Source: Yahoo Finance

Return Calculation (Log Returns)

$$r_t = 100 \times \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where P_t = closing price at time t

Methodology Flow

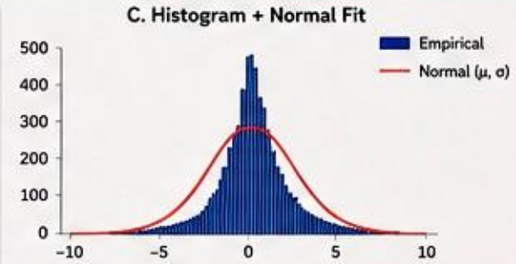
Prices → Log Returns → Descriptive Stats → Distribution Analysis → Normality Test → Risk Implications

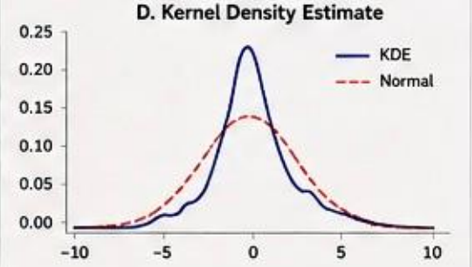
Why It Matters

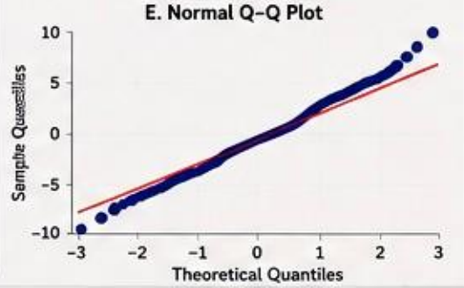
Distributional assumptions directly influence VaR, Expected Shortfall, portfolio optimization, risk management and financial decision-making.

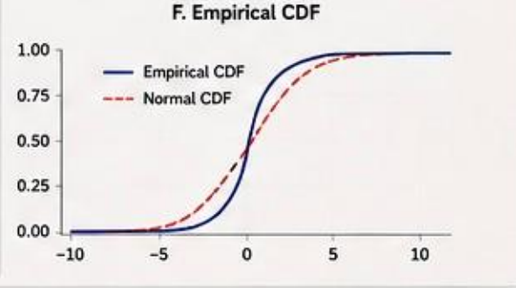
N = 4,030	0.046%	1.083%	-0.402	6.742	JB p-value < 0.001
Observations	Mean Return	Std. Deviation	Skewness	Kurtosis	Reject Normality

4. EMPIRICAL DISTRIBUTION ANALYSIS









5. Quantile Comparison (Empirical vs Normal Theory)

Quantile	Empirical (Historical)	Normal Theoretical	Difference
1%	-2.87%	-2.38%	-0.49%
5%	-1.71%	-1.65%	-0.06%
25%	-0.66%	-0.73%	+0.07%
50% (Median)	0.04%	0.05%	-0.01%
75%	0.75%	0.82%	-0.07%
95%	1.96%	1.75%	+0.21%
99%	3.36%	2.58%	+0.78%

6. Extreme Returns (Top 5)

Rank	Date	Return (%)
1	2020-03-16	-11.98
2	2020-03-12	-9.51
3	2020-03-09	-7.60
4	2008-10-15	-7.29
5	2020-06-11	5.66

Extreme events occur more frequently than predicted by a normal distribution.

7. NORMALITY TEST (JARQUE–BERA)

$$JB = \frac{n}{6}\left(s^2 + \frac{(K-3)^2}{4}\right)$$

Where s = skewness, K = kurtosis, n = observations

Hypotheses

H_0: Returns are normally distributed
H_1: Returns are not normally distributed

Results (Example)

JB Statistic: 3,245.71
p-value: < 0.001
Decision: Reject H_0

8. KEY FINDINGS

- Returns exhibit negative skewness, indicating larger negative moves than positive moves.
- Kurtosis is significantly greater than 3 → heavy tails.
- Jarque–Bera test strongly rejects the normality assumption.
- KDE shows a sharper peak at the center and fatter tails than the normal distribution.
- Q-Q plot shows strong deviation in both tails.
- Empirical quantiles in the tails are more extreme than normal theory.
- Extreme market events occur more frequently than what a normal model predicts.

9. FINANCIAL IMPLICATIONS

- Normality assumption underestimates extreme risk.
- VaR based on normal distribution may be misleading.
- Risk models should incorporate fat tails and asymmetry.
- Better risk measurement → better financial decisions.
- Importance for stress testing & capital allocation.

10. LIMITATIONS

- Results are sample-period dependent.
- Historical patterns may not persist in future.
- Normality tests are sensitive to large samples.
- KDE depends on bandwidth selection.
- Unconditional analysis does not capture volatility dynamics.

11. CONCLUSION

The empirical evidence strongly suggests that daily S&P 500 returns are not normally distributed. Returns exhibit skewness, excess kurtosis and heavy tails. These findings have important implications for financial risk modeling, particularly for estimating tail risk and extreme market events.

12. THE BIG TAKEAWAY

"Financial returns may look simple around the mean — but the tails tell a very different story."

Distributional assumptions → Risk estimates → Decisions

13. REFERENCES

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